Studies of NLL and RG-improved BFKL evolution with saturation effects

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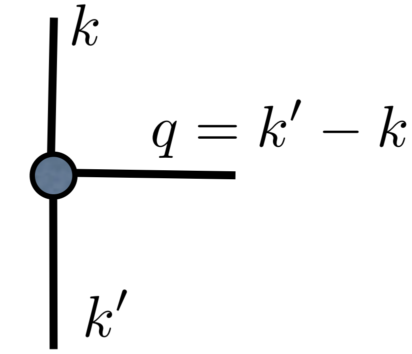
Main points and outline

- Small-x evolution with BFKL
- Probing non-linear evolution via saturation boundary
- Problems of NLL evolution
- Improving small-x evolution with DGLAP: RG improved BFKL
- Results for fixed and running coupling NLL evolution
- RG improved BFKL and the dip
- RG improved non-linear evolution

Small-x evolution with BFKL

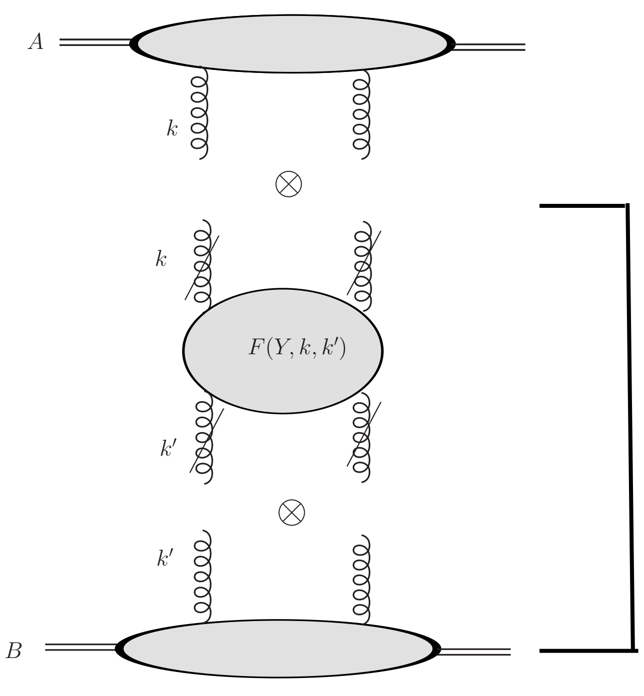
- BFKL is the main ingredient of small-x studies. Eq for "Gluon Green's function" G in Regge limit:

$$\partial_{\ln\zeta} G(\zeta, k, k_0) = \int d^2k' K(k,k') \otimes G(\zeta, k', k_0)$$



- Kernel known to NLL order: $K(k',k) = \alpha_s K_0(k',k) + \alpha_s^2 K_1(k',k)$

Cross sections calculated as: $\sigma^{AB} = \int d^2k_1 d^2k_2 \Phi_A(k_1) G(\zeta, k_1, k_2) \Phi_B(k_2)$



$$F(\zeta, k_1) = \int d^2k_2 G(\zeta, k_1, k_2) \Phi(k_2)$$

$$\partial_{\ln\zeta} F(\zeta, k_1) = \int d^2k' K(k_1, k') \otimes F(\zeta, k')$$

Non-linear Evolution from "saturation boundary"

- Form of generic non-linear evolution eq:

$$\partial_{\ln \zeta} F = K_{BFKL} \otimes F_\zeta + \Gamma_2 \otimes F_\zeta^2 + \Gamma_3 \otimes F_\zeta^3 + \dots$$

In coordinate space, NLL BK: $\partial_\zeta \mathcal{N} = K_{BFKL} \otimes \mathcal{N} + K_2 \otimes (\mathcal{N}^2 - \mathcal{N}^3)$

$$1 - \mathcal{N}(\zeta, r) = \int \frac{d^2k}{(2\pi)^2} e^{ik\cdot r} F(\zeta, k)$$

Non-linear parts can be rather complicated and very difficult to deal with even numerically.

Main properties of evolution driven by linear kernel. Non-linear term provides a "cut-off" to suppress strong linear growth.

Thus effectively much simpler to:

$$\partial_Y \mathcal{A} = K_{BFKL} \otimes \mathcal{A} + \text{nonlinear} \quad \Longrightarrow \quad \partial_Y \mathcal{A} = K_{BFKL} \otimes \mathcal{A} + \text{boundary}$$

where $\mathcal{A} = \{\mathcal{N}, \mathcal{F}\}$ and $Y = \ln 1/\zeta$

Problems of NLL evolution and (a) cure

LL eigenvalue: $$\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma) \sim \frac{1}{\gamma} + \frac{1}{1-\gamma}$$

NLL eigenvalue: (symmetric)

$$\chi_1(\gamma) \sim \underbrace{-\frac{1}{2\gamma^3} - \frac{1}{2(1-\gamma)^3}}_{\text{Energy scale (kin. const)}} + \underbrace{\frac{A_1(0)}{\gamma^2} + \frac{A_1(0) - b}{(1-\gamma)^2}}_{\text{DGLAP}}$$

b ← Running coupling

DGLAP $A_1(0) = -\frac{11}{12}$

Negative poles cause instability of solution. $\mathcal{F}(\zeta, k)$ and σ turns negative and oscillate.

Same terms appear in NLL BK as well, thus we can expect problems...

Cure: Subtract these negative poles and demand agreement with full DGLAP and kin. constraint. Leads to "RG-improved" BFKL:

$$\omega = \alpha_s \chi$$

$$\chi_{\text{resum}} = \underbrace{\chi_0(\gamma, \omega)}_{\text{LL w kin.const.}} + \underbrace{\chi_{coll}(\gamma, \omega)}_{\text{DGLAP terms w kin. const.}} + \underbrace{\tilde{\chi_1}(\gamma)}_{\text{Subtractions}}$$

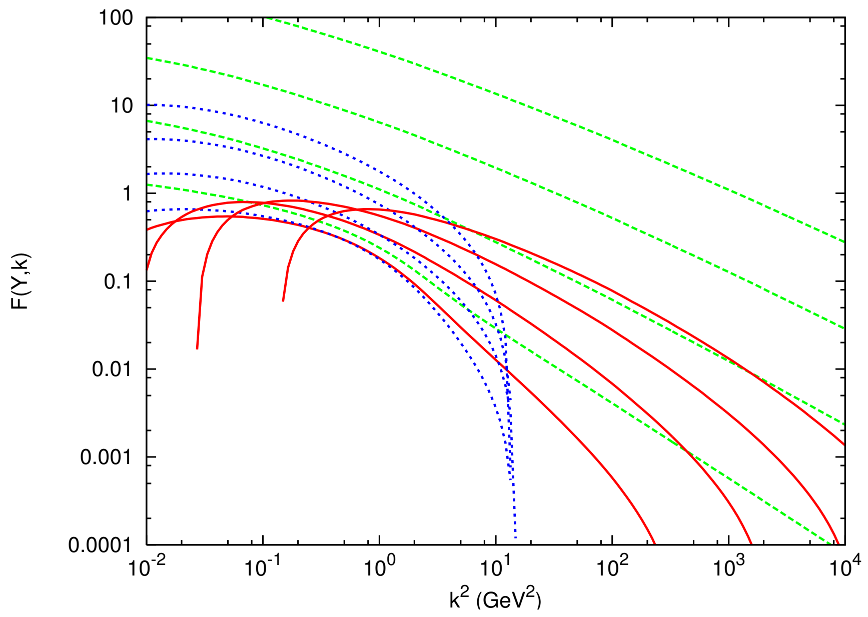
Different resummation procedures exist:

CCSS, ABF, TW...

All consistent with each other.
We use here CCSS (B) formalism.

No such procedure yet for BK but we can here study RG improved BFKL with saturation boundary: $\partial_Y \mathcal{F} = K_{\text{resum}} \otimes \mathcal{F} + \text{boundary}$

Appetizer: NLL evolution with fixed coupling



$\bar{\alpha}_s = 0.2$

Green: LL

Red: NLL asymmetric scale

Blue: NLL symmetric scale

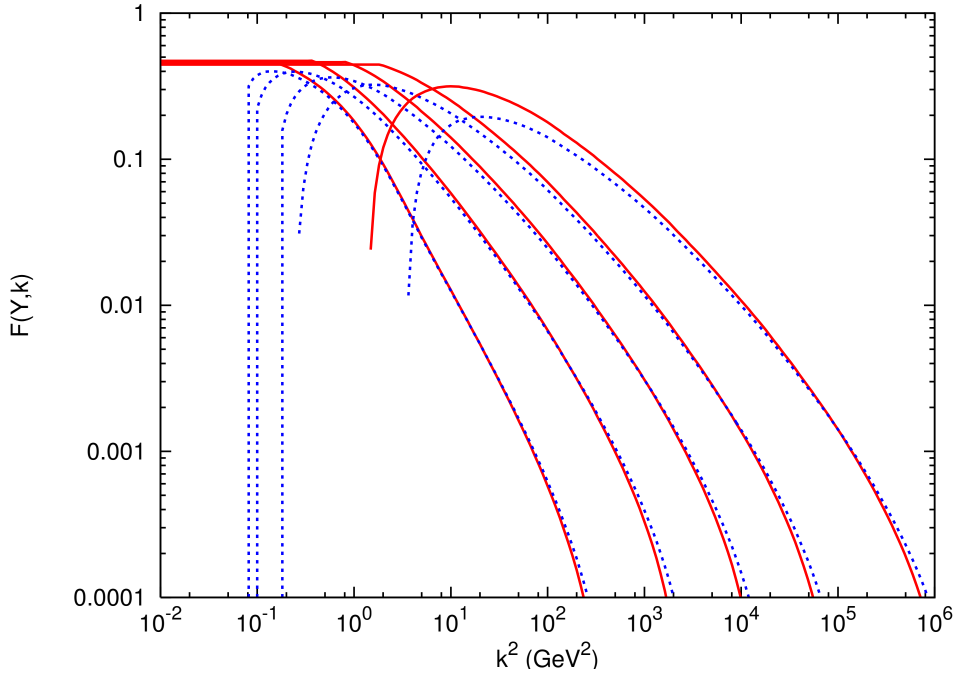
$Y = 2, 6, 10, 14$

Relevant choice for DIS and saturation problem

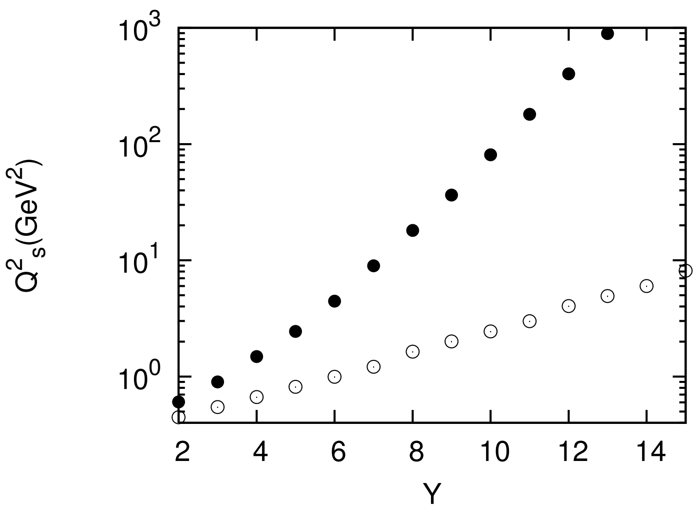
With two different saturation boundaries

Red: Frozen

Blue: Absorptive



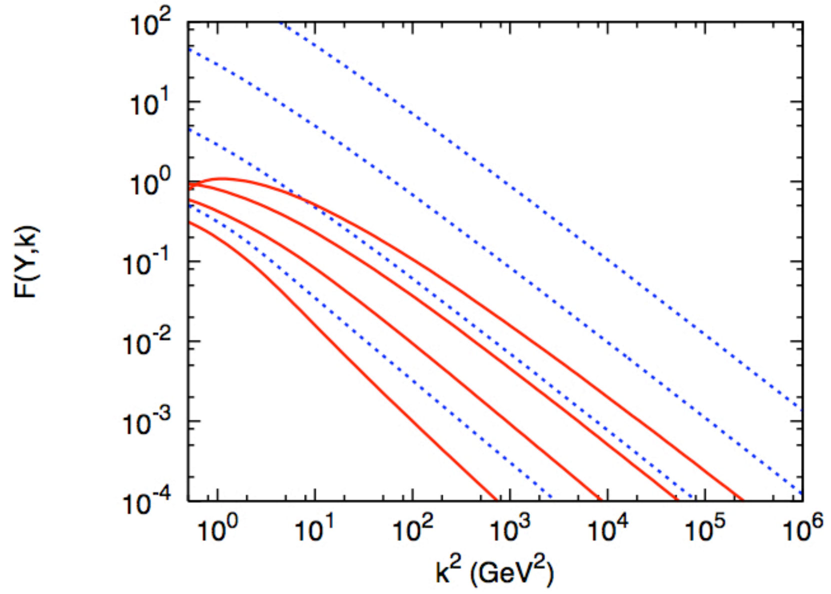
Saturation Scale:
LL and NLL



Note: Solution at small k still unstable, not cured by saturation. High k also unstable as in linear case

Full NLL evolution with running coupling

Better behavior at high k due to running coupling. However, HUGE uncertainty on scale choice: some choice ok but some terrible.

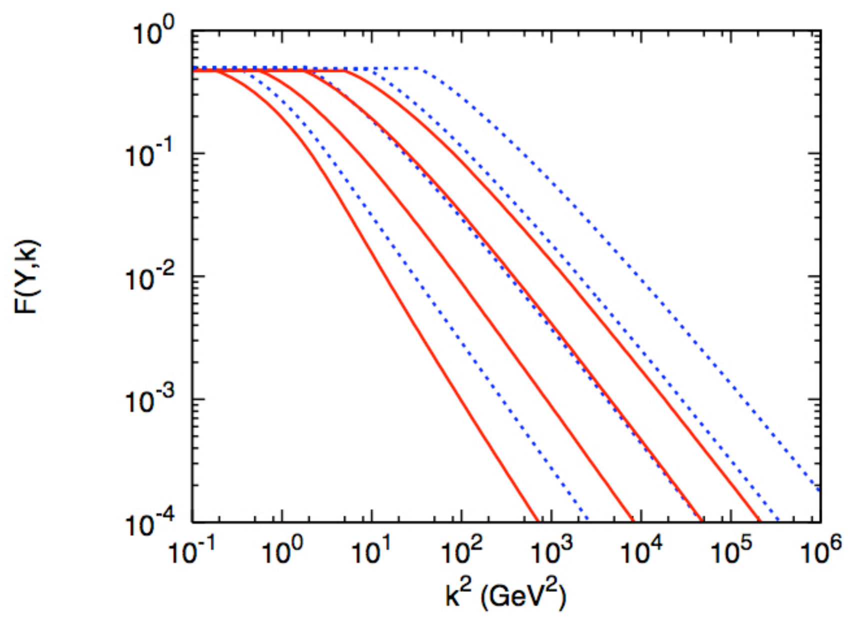


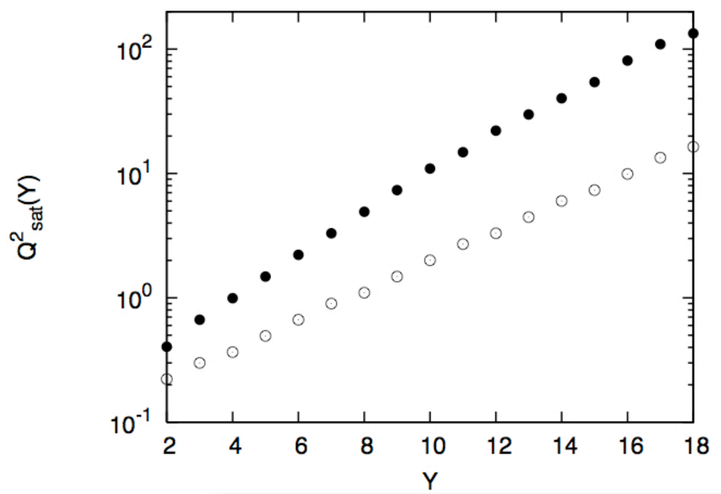
Choice: $\alpha_s(q)K_0 + \alpha_s^2(max(k,k'))K_1$ $\quad Y = 2, 6, 10, 14$

Blue: rcLL
Red: rcNLL

Anti-collinear pole still there and solution goes negative at low k

Apply boundary to regulate low k region





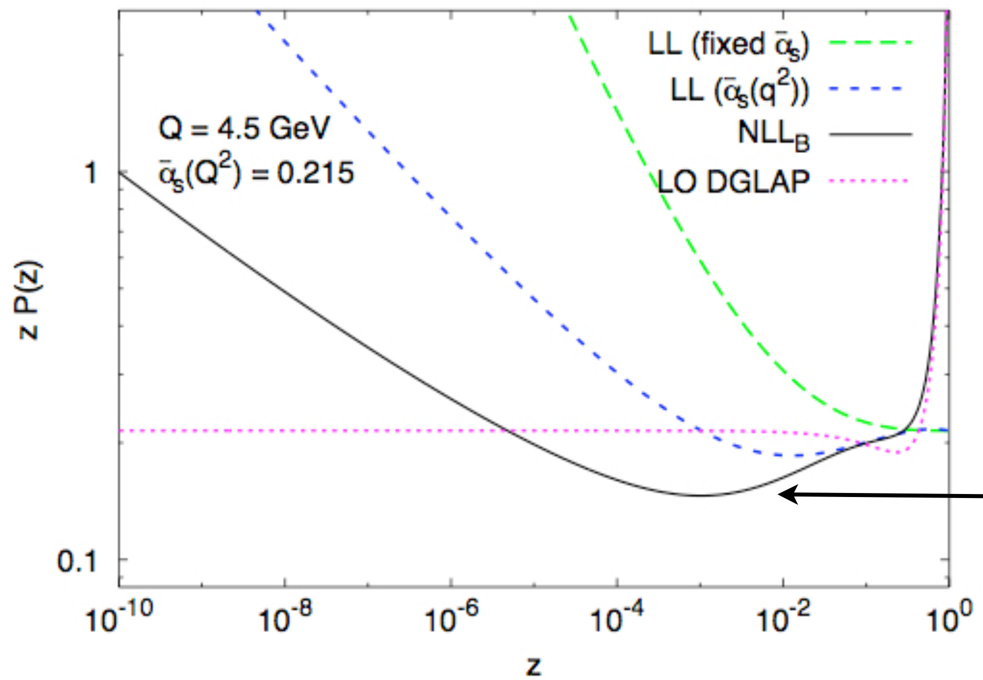
Large difference in sat. scale between rcLL and rcNLL evolution

Different boundary condition gives very similar results, again large difference LL vs NLL.

These results look ok for rcNLL, but again this is highly dependent on precise choice of scale. Thus NLL evolution, linear or non-linear, is very unstable!! Resummation needed.

Resumed evolution: The dip and implications on saturation

Plot from CCSS: hep-ph/0307188



"dip"

Splitting function extracted from resummed evolution has characteristic "dip" structure

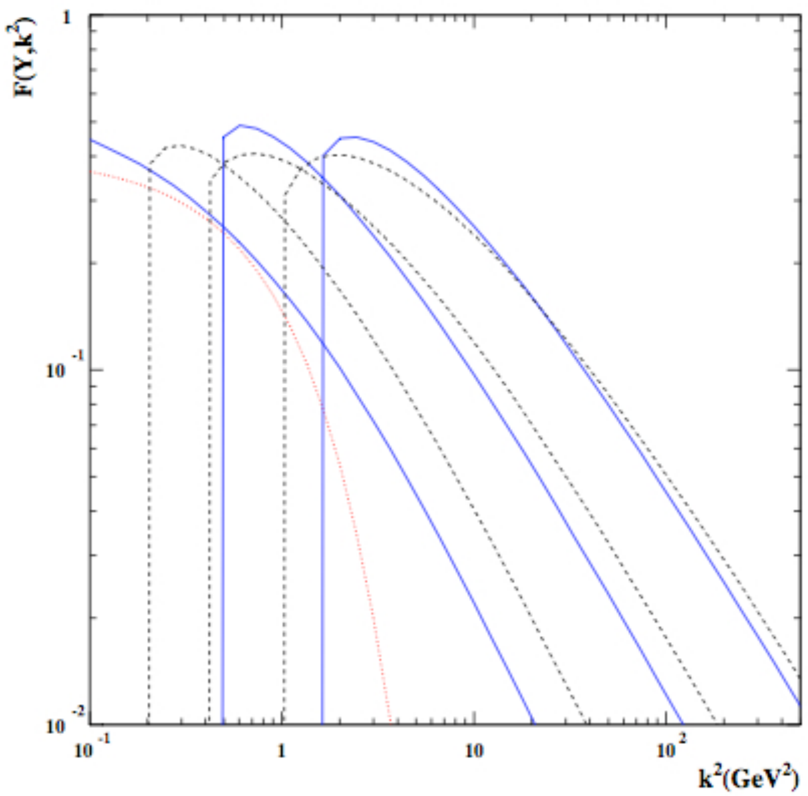
Originates from interplay between positive LL terms and negative NLL terms. Also found in ABF and TW.

Precise position of dip depends on α_s and depth on α_s and n_f

Consequently it takes "time" for small x evolution to fully set in.
Similar phenomenon observed in "unified BFKL-DGLAP" evolution and in CCFM.

Obviously *important* for physics of saturation and has implications for phenomenology.

RG improved evolution: Linear and non-linear

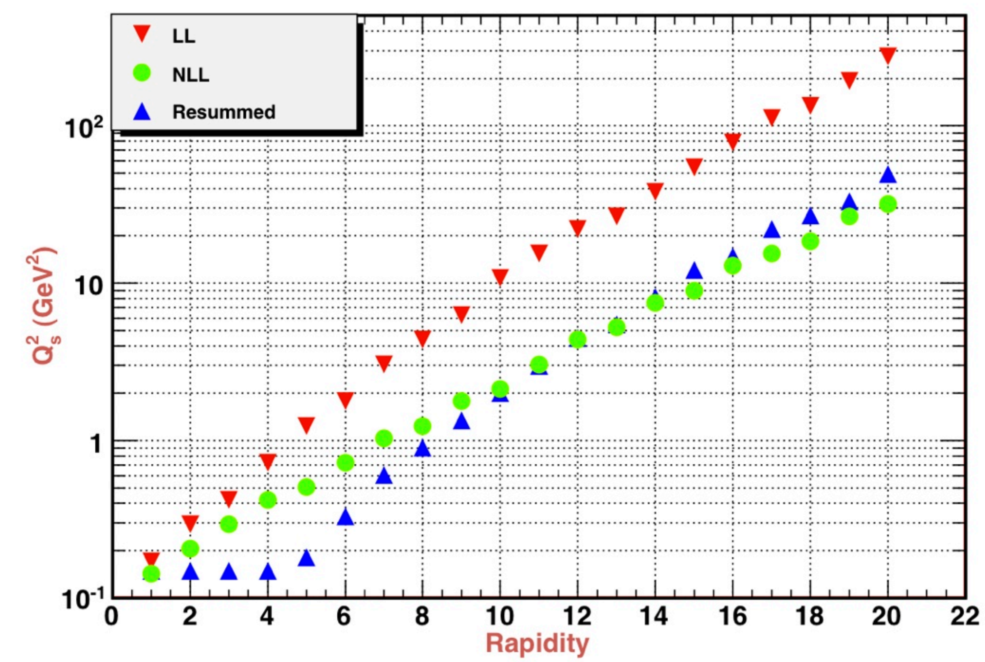


$$\alpha_s(q)K_0 + \alpha_s^2(max(k,k'))K_1$$

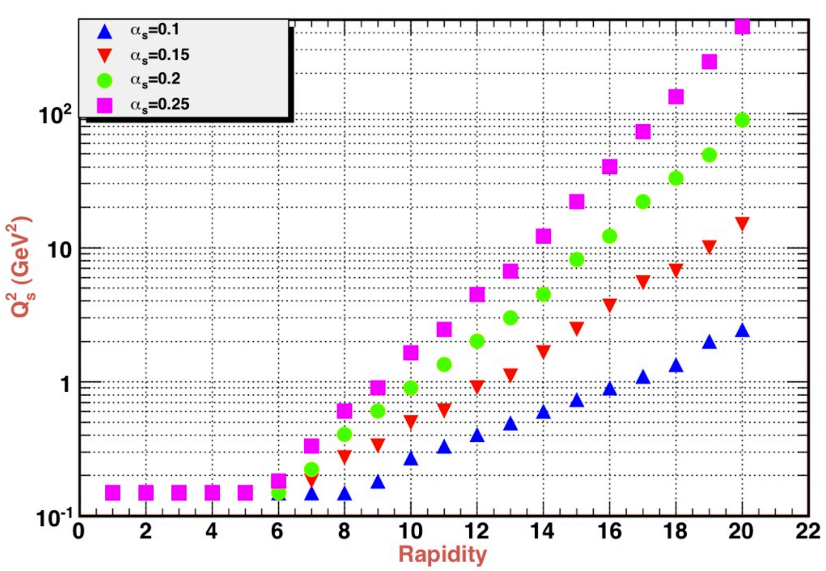
Blue: Resummed
Black: NLL

$$Y = 4, 8, 12$$

Sat. scale from
resummed evolution



Fixed coupling. Variation of
dip with coupling



Existence of dip clearly delays growth of Q_s At which Y exactly the growth sets in depends on precise parameters, initial condition etc.
We can answer this only after serious application to phenomenology.

Nonetheless the structure is there and is important!